Complex numbers and the Erdös - Mordell inequality

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Submitted to publication.

Key Words : Complex numbers, dot product, cross product, polar form, cartesian form, triangle inequality, Erdös-Mordell inequality.

MSC: Primary: 51M04, Secondary: 30A10, 15A72.

Summary

Let ΔABC be a triangle and O a point in it. Consider a coordinate system with O as its origin and A is on the positive direction of the X-axis, B is in the first or second quadrant, and C is in the third or fourth quadrant. We associate to the vertices A, B, and C their corresponding complex numbers

$$\begin{array}{lll} A = |A|e^{i\theta_A} & , & \theta_A = 0, \\ B = |B|e^{i\theta_B} & , & \theta_B \in (0, \pi), \\ C = |C|e^{i\theta_C} & , & \theta_C \in [\pi, 2\pi), \quad \text{and} \quad \theta_C - \theta_B \in (0, \pi). \end{array} \tag{0.1}$$

The conditions on the angles imply that O is in the triangle. We also associate to the foot of each perpendicular from O to the sides their corresponding complex number : P for the side joining the vertices B and C, Q for the side joining the vertices C and A, and R for the side joining the vertices A and B. With this notation, the Erdös-Mordell inequality is

$$2(|P| + |Q| + |R|) \leq |A| + |B| + |C|. \tag{0.2}$$

In this short note we use dot and cross products of complex numbers, operations usually introduced in textbook as a curiosity, see [3, 4]. For the history of this inequality and references about other proofs, see [1].

References

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