On the modified Newton's method
for multiple root

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Summary

The modified Newton's method can be used for solving an equation for a root of known multiplicity greater than one [6] (see also [1, 5]). In this paper we present general convergence results about this method an a way to increase its order of convergence. As a special case we recover the convergence results for the Newton's method used to find a simple root and the usual way to increase its order of convergence, known as the Schröder's method of the second kind [4]. However, since the modification to the Newton's method depends on the multiplicity of the root, and this multiplicity is often not known a priori, the results presented here are more of theoretical interest than of practical importance.

In Section 2, after some preliminaries we state and prove two lemmas which are fundamental for the next sections of this paper. Section 3 is devoted to the convergence results of the modified Newton's method, while a method to increase the order of convergence is explained in Section 4.

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