On the Chebyshev-Halley family of iteration functions

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Summary

The Chebyshev-Halley family of iteration functions (IFs) to solve \( f(x) = 0 \) has been introduced by Werner [12]. It can also be found in [1] and [5], as reported in [11]. Each member of this family is obtained as an improvement of the Newton’s IF, depends on a real parameter \( \beta \), and can be written as

\[
G_\beta(x) = x - \frac{f(x)}{f'(x)} \left[ \frac{1 - (\beta - 1/2)L_f(x)}{1 - \beta L_f(x)} \right]
\]

where \( L_f(x) = \frac{f(x)f''(x)}{[f'(x)]^2} \). These IFs are of order 3 when we look for an \( \alpha \in \mathbb{R} \) such that \( f(\alpha) = 0 \) and \( \alpha \) is a simple root of \( f(x) \).

In this paper we obtain this family of IFs from a linear combination of two Newton’s IFs. We will also see how to modify the parameter \( \beta \), and express it as a function of \( x \), to obtain an IF of order 4.

We also consider the best parameter \( \beta \) for the \( n \)-th root computation problem. For this problem the best parameter depends only on \( n \). In particular, when we compare the Halley (\( \beta = 1/2 \)) and Super-Halley (\( \beta = 1 \)) IFs, we show that Super-Halley IF is the best method to compute the \( n \)-th root for \( n = 2, 3, 4 \), Halley IF is the best method for \( n \geq 6 \), and they are equivalent for \( n = 5 \). Both are better than the Chebyshev (\( \beta = 0 \)) IF.

References