Exponential growth model : from horizontal to linear asymptote

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Summary

Growth functions with horizontal asymptotes as Gompertz, Logistics, Richards (generalized Logistics), Bridge or Michaelis-Menten are commonly and widely used in various fields, e.g. agriculture [8, 9, 11, 12, 15], biology [7, 16, 25, 26], economy [21], fishery [23], forestry [18, 28, 29], hydrology [1, 2, 4, 5, 6, 13, 17, 27], medicine [14], and other areas of applied research [19, 20, 22]. These growth functions are characterized by an horizontal asymptote and a single inflection point [3].

In some applications, for example in hydrology, it could be usefull to have growth functions with one inflection point and an increasing concave curvilinear asymptote, and even an oblique asymptote. In this paper we present one such situation. We modify an exponential function having a natural horizontal asymptote to obtain a model having an increasing concave curvilinear asymptote and, for the limiting case, having an increasing linear asymptote. The model retains the single inflection point property. We present examples on synthetic data and also apply the model to a real situation in hydrology.

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