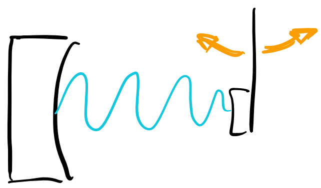
→ Optomechanical Hamiltonian.



$\hat{b}$: mechanics

$\hat{a}$: optics.

$$\hat{H} = \hbar\omega(\hat{x})\hat{a}^\dagger\hat{a} + \hbar\Omega\,\hat{b}^\dagger\hat{b}$$

$$\hat{x} = x_{ZPF}\left(\hat{b}^\dagger + \hat{b}\right).$$

Typical cavity: $\omega(\hat{x}) = \omega_c\left[1 - \frac{\hat{x}}{L} + \mathcal{O}(\hat{x}^2)\right]$.

↳ 1^{st} order ⇒ new term of the form $-\hat{F}\hat{x}$.

with $\hat{F} = \frac{\hbar\omega_c}{L}\hat{a}^\dagger\hat{a}$.

$$\boxed{\hat{H} = \hbar\omega_c\hat{a}^\dagger\hat{a} + \hbar\Omega\,\hat{b}^\dagger\hat{b} - \hbar\frac{\omega_c}{L}\hat{a}^\dagger\hat{a}\,\hat{x}}$$

We define de coupling strength:

$$g_0 = \omega_c\frac{x_{ZPF}}{L}$$

More generally: $g_0 = -\frac{\partial\omega}{\partial x}x_{ZPF}$.

i.e. by how much does the zero point fluctuation changes the cavity resonnance

→ What can we do with this interaction

↳ cooling / heating.

↳ first classical picture:

How can we use the cavity to cool?

Interaction → Force $\propto$ # photon $\times$ x

⇒ optical spring.

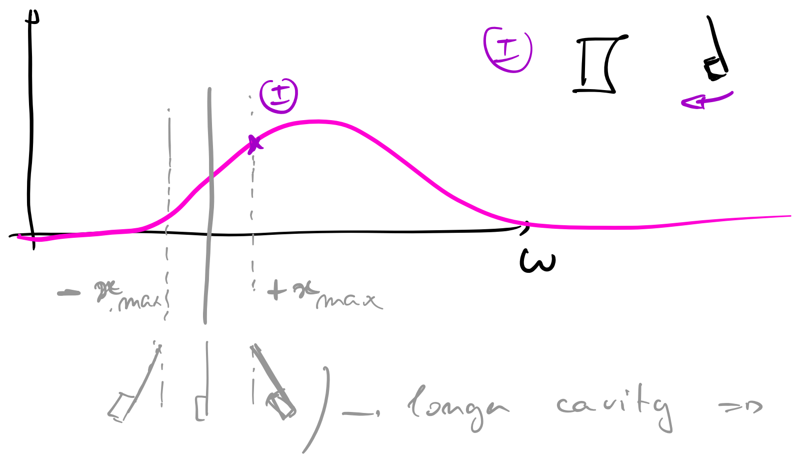
Mechanical spring constant:

$$D = D_0 - \frac{dF}{dx}.$$

$F \to kx$ form ⇒ conservative

One needs to account for the lag in the cavity.

Q-factor of cavity ⇒ # photon change at rate $1/K$



→ longer cavity ⇒ ω_c ↘

→ more photon when coming back
⇒ slow down
— fewer photon on way back
⇒ we don't re-accelerate.

↳ Cooling happens because of lag.
the higher the lag, the better.

"Quantum picture":

In the rotating frame: $\hat{U} = e^{i\omega_L \hat{a}^\dagger \hat{a} t}$.

$$\hat{H} = \hbar\Delta \hat{a}^\dagger \hat{a} + \hbar\Omega \hat{b}^\dagger \hat{b} \underbrace{- \hbar g_0 \hat{a}^\dagger \hat{a} (\hat{b}^\dagger + \hat{b})}_{H_{int}}.$$

$$\Delta = \omega_L - \omega_c.$$

Linearisation: equivalent to applying displacement operator

Coherent amplitude: $\langle \hat{a} \rangle = \bar{\alpha}$

$$\hat{a} = \bar{\alpha} + \delta\hat{a}$$

$$H_{int} = -\hbar g_0 (\bar{\alpha} + \delta\hat{a})^\dagger (\bar{\alpha} + \delta\hat{a})(\hat{b}^\dagger + \hat{b}).$$

$$= \overbrace{-\hbar g_0 |\bar{\alpha}|^2}^{\text{shift}} (\hat{b}^\dagger + \hat{b}) - \hbar g_0 (\bar{\alpha}^* \delta\hat{a} + \alpha\, \delta\hat{a}^\dagger)(\hat{b}^\dagger + \hat{b})$$
$$- \hbar g_0 \underbrace{\delta\hat{a}^\dagger \delta\hat{a}}_{\simeq 0} (\hat{b}^\dagger + \hat{b}) .$$

For large-int field leading terms in $\bar{\alpha}$ are kept.

$-\hbar g_0 |\bar{\alpha}|^2 (\hat{b}^\dagger + \hat{b}) \to$ average shift of $\hat{x}$ du radiation pressure.

Can be hidden by taking new reference for $\hat{x}$.

It could be a problem if this shift is large compared to Ken non-linearity of mechanics.

$$\boxed{\hat{H}_{int} \simeq \underbrace{-\hbar g_0 \sqrt{\bar{n}_c}}_{} (\delta\hat{a}^\dagger + \delta\hat{a})(\hat{b}^\dagger + \hat{b}) .}$$

$\hat{X}\,\hat{X}$ type interaction.

$\hookrightarrow$ photon enhanced coupling $g_0 \sqrt{\bar{n}_c}$

Favourable regimes:

$\hookrightarrow$ $g > \kappa$ $\Rightarrow$ strong coupling

$\Rightarrow$ interaction faster then losses (in general $\Gamma_m \gg \kappa$)

2, $g_0 > \kappa \Rightarrow$ single-photon strong coupling.

interaction with 1 photon sufficiently strong $\Rightarrow$ non-linear optomechanics.

→ Different RWA for different detunings:

$\Delta \simeq -\Omega$, red side band:

$$H_{int} = -\hbar g \left(\delta \hat{a}^+ \hat{b} + \delta \hat{a} \hat{b}^+ \right).$$

→ cooling
→ state transfer
) "beam-splitter interaction"

$\Delta \simeq +\Omega$, blue side band:

$$H_{int} = -\hbar g \left(\delta \hat{a}^+ \hat{b}^+ + \delta \hat{a} \hat{b} \right).$$

→ amplification
→ entangler, squeezer
) "two-mode squeezer"

$\Delta \simeq 0$, on resonance

$$H_{int} = -\hbar g \left(\delta \hat{a}^+ + \delta \hat{a} \right)\left(\hat{b}^+ + \hat{b} \right)$$

→ position shift $\Rightarrow$ phase shift
→ QND measurement of $\delta \hat{a}^+ + \delta \hat{a}$

→ Equation of motion.

↳ easiest way is through q Langevin equations.

Both degree of freedom $\hat{a}$, $\hat{b}$ are coupled to thermal bath (Fluctuations).
Treating the bath in Born-Markov approximation.

q Langevin in short:

$$\hat{H} = \hat{H}_{syst} + \hat{H}_{syst-bath}.$$

Harmonic oscillators:

$$\hat{H}_{syst} = \frac{\hat{p}^2}{2m} + \hat{V}(\hat{q})$$

coupling

$$\hat{H}_{syst-bath} = \sum_j \left[\frac{\hat{p}_j}{2m_j} + \frac{k_j}{2} (\hat{q}_j - \hat{q})^2 \right]$$

Heisenberg eq°:

$$\dot{\hat{O}} = \frac{1}{i\hbar}[\hat{O}, H_{syst}] + \frac{1}{i\hbar}[\hat{O}, \hat{H}_{syst-bath}]$$

error

$$= \frac{1}{i\hbar}[\hat{O}, \hat{H}_{syst}] - \frac{1}{2i\hbar}\sum_j k_j \left\{ [\hat{O}, \hat{q}] \cancel{\hat{q}_j}, \hat{q}_j - \hat{q} \right\}_+$$

$$\{A, B\}_+ = AB + BA.$$

Markovian & Langevin:

$$\dot{\hat{O}} = \frac{1}{i\hbar}\left[\hat{O}, \hat{H}_{syst}\right] + i\sqrt{2\gamma}\left[\hat{O}, \hat{P}\right]\hat{P}_{in}(t) + \frac{1}{2i\varphi}\left\{\left[\hat{O}, \hat{\varphi}\right], \dot{\hat{\varphi}}(t)\right\}_+$$

$$\varphi = \frac{\Omega}{\gamma}.$$

For optomechanics:

$$\dot{\hat{a}} = -\frac{\kappa}{2}\hat{a} + i(\Delta - G\hat{x})\hat{a} + \sqrt{\kappa_{ex}}\,\hat{a}_{in} + \sqrt{\kappa_0}\,\hat{f}_{in}$$

$$\dot{\hat{b}} = -\frac{\Gamma}{2}\hat{b} - i\Omega\hat{b} + ig_0\hat{a}^\dagger\hat{a} + \sqrt{\Gamma}\,\hat{b}_{in}.$$

$$G = \frac{g_0}{x_{ZPF}} \qquad \frac{1}{\kappa} = \frac{1}{\kappa_{ex}} + \frac{1}{\kappa_0}.$$

With the noise correlators:

$$\langle \hat{a}_{in}(t)\,\hat{a}^\dagger_{in}(t')\rangle = \delta(t-t')$$
$$\langle \hat{a}^\dagger_{in}(t)\,\hat{a}_{in}(t')\rangle = 0$$

empty cavity !

$$\langle \hat{b}_{in}(t)\,\hat{b}^\dagger_{in}(t')\rangle = (n_{th}+1)\,\delta(t-t')$$
$$\langle \hat{b}^\dagger_{in}(t)\,\hat{b}^\dagger_{in}(t')\rangle = n_{th}\,\delta(t-t')$$

empty cavity $\Leftrightarrow$ $\frac{k_B T}{\hbar\omega_c} \simeq 0.$

generally Ω small $\Rightarrow$ $\frac{k_B T}{\hbar\Omega} \gg 1.$ $(n_{th} \gg 1)$

→ Classical version:

$$\langle \hat{a}(t) \rangle = \alpha(t)$$
$$\langle \hat{x}(t) \rangle = x(t).$$
$$\dot{\alpha} = -\kappa/2\, \alpha + i(\Delta - Gx)\alpha + \sqrt{\kappa_{ex}}\, \alpha_{in}$$
$$m_{eff}\, \ddot{x} = -m_{eff}\, \Omega^2 x - m_{eff}\, \Gamma \dot{x} + \hbar G |\alpha|^2$$

Full Quantum Langevin, in practice not solvable. To access the q fluctuation, one can apply the same linearization as for μ: $\hat{a} = \alpha + \delta\hat{a}$. $\alpha = \sqrt{\bar{n}_c}$.

→ Linearized Classical Langevin:

$$\delta\dot{\alpha} = \left(i\Delta - \frac{\kappa}{2}\right)\delta\alpha + iG\bar{\alpha}x$$
$$m_{eff}\ddot{x} = -m_{eff}\Omega^2 x - m_{eff}\Gamma\dot{x} + \hbar G\left(\bar{\alpha}^*\delta\alpha + \bar{\alpha}\delta\alpha^*\right)$$

Fourier transform:

$$-i\omega\,\delta\dot{\alpha} = \left(i\Delta - \frac{\kappa}{2}\right)\delta\alpha + iG\bar{\alpha}x$$
$$-m_{eff}\omega^2\ddot{x} = -m_{eff}\Omega^2 x + i\omega m_{eff}\Gamma x + \hbar G\left(\bar{\alpha}^*\delta\alpha + \bar{\alpha}\delta\alpha^*\right)$$

$G = 0 \Rightarrow$ mechanical susceptibility,

- $\chi^{-1}_{xx,0}(\omega) = m_{eff}\left[(\Omega^2 - \omega^2) - i\Gamma\omega\right].$

Coupling to the cavity $(G \neq 0)$

$$\Rightarrow \chi_{xx}^{-1} = \chi_{xx,0}^{-1}(x) + \underbrace{\Sigma(\omega)}_{\text{self energy}}$$

the mechanical susceptibility is changed by the radiation pressure. κ is the cavity lag.

Solving for $\bar{\alpha}$ and $\delta\alpha$ we get:

$$\Sigma(\omega) = 2 m_{eff} \Omega g^2 \left\{ \frac{1}{(\Delta+\omega) + i\kappa/2} + \frac{1}{(\Delta-\omega) - i\kappa/2} \right\}.$$

The real and imaginary part have $\neq$ roles.

$\hookrightarrow \chi_{xx}(\omega)$ → real $\Rightarrow$ mech. freq.
→ imag $\Rightarrow$ mech damping

$$\Sigma(\omega) = m_{eff}\, \omega \left[2\delta\Omega(\omega) - i\Gamma_{opt}(\omega) \right].$$

$$\hookrightarrow \chi_{xx}^{-1}(\omega) = m_{eff}\left[\Omega^2 + 2\omega\delta\Omega - \omega^2 - i\omega\left[\Gamma + \Gamma_{opt}(\omega)\right]\right]$$

$$\delta\Omega = g^2 \frac{\Omega}{\omega}\left[\frac{\Delta+\omega}{(\Delta+\omega)^2 + \kappa^2/4} + \frac{\Delta-\omega}{(\Delta-\omega)^2 + \kappa^2/4} \right]$$

$$\Gamma_{opt} = g^2 \frac{\Omega}{\omega}\left[\frac{\kappa}{(\Delta+\omega)^2 + \kappa^2/4} - \frac{\kappa}{(\Delta-\omega)^2 + \kappa^2/4} \right]$$

→ Optical spring : $\delta\Omega$ @ $\omega = \Omega$.

$$\delta\Omega = g_0^2 \bar{n}_c \left[\frac{\Delta+\Omega}{(\Delta+\Omega)^2 + \kappa^2/4} + \frac{\Delta-\Omega}{(\Delta+\Omega)^2+\kappa^2/4} \right].$$

→ Optical damping :

$$\Gamma_{opt} = g_0^2 \bar{n}_c \left[\frac{\kappa}{(\Delta+\Omega)^2+\kappa^2/4} - \frac{\kappa}{(\Delta-\Omega)^2+\kappa^2/4} \right].$$

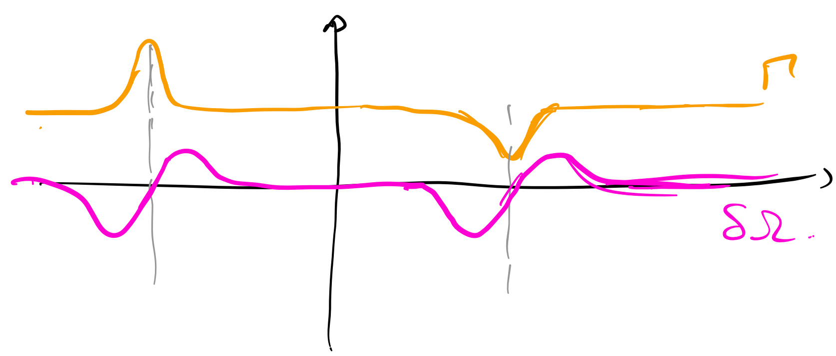
$$\Gamma_{eff} = \Gamma + \Gamma_{opt}.$$

Drawing the different ones.

Resolved sideband

Unresolved ———.

$\kappa \ll \Omega$



Why $\Gamma_{eff} \gg \Gamma \Rightarrow$ cooling.

Extra damping from a "cold" cavity.

$$\Gamma_{opt}\big|_{\kappa \ll \Omega} = \frac{4 g_0^2}{\kappa} \bar{n}_c.$$

cooperativity : $C_0 = \frac{4 g_0^2}{\kappa \Gamma}$.

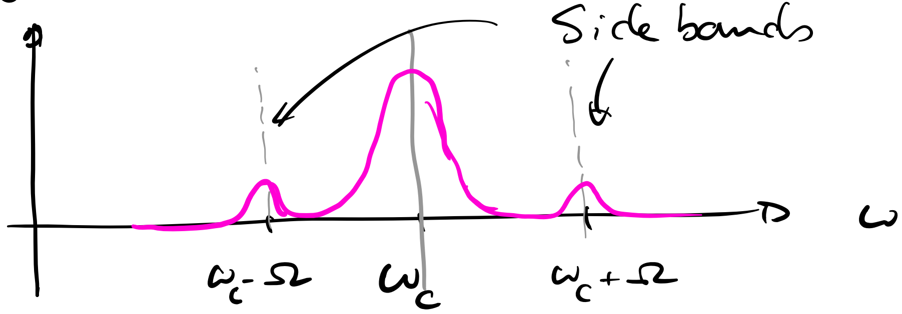
$C = \Gamma_{opt}/\Gamma$

↳ Back with the hands:

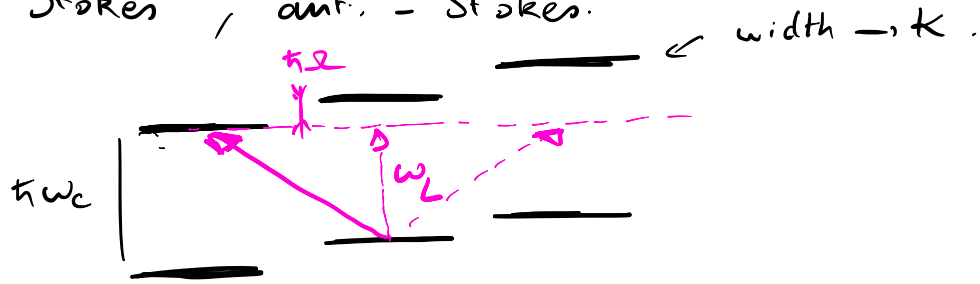
cavity lag ⇒ dissipative force.

Why do we want $\kappa \ll \Omega$ (resolved sidebands)

Cavity + mechanics response:



↳ we can understand this as Stokes, anti-Stokes.



↳ Cooling limits:

"naive" picture: $T_{final} = T_{init} \dfrac{\Gamma}{\Gamma + \Gamma_{opt}}$

$\Gamma_{opt} \xrightarrow[\bar{n}_c \to \infty]{} \infty \Rightarrow T_{final} \longrightarrow 0.$

We are missing shot noise again

two processes:

$$\Gamma_{n\to n-1} = n A^-$$
$$\Gamma_{n\to n+1} = (n+1) A^+$$

Γ_{opt} simply: $\Gamma_{opt} = A^- - A^+$.

Evolution of phonon #: $\bar{n}_p$.

$$\dot{\bar{n}}_p = (\bar{n}_p + 1)(A^+ + A^+_{th}) - \bar{n}_p(A^- + A^-_{th}).$$

$A^+_{th} = \bar{n}_{th}\Gamma$ if $\bar{n}_{th}=0$ can't gain

$A^-_{th} = (\bar{n}_{th}+1)\Gamma$

@ equilibrium: $\dot{\bar{n}}_p = 0$.

$$\bar{n}_f = \frac{A^+ + \bar{n}_{th}\Gamma}{\Gamma_{opt}+\Gamma} < \bar{n}_{min} = \frac{A^+}{\underbrace{A^- - A^+}_{\Gamma_{opt}}}$$

We can calculate $A^\mp$ with the Fermi golden rule:

Forcing term: radiation pressure from cavity field.

$\hat{H}_{int} = -\hat{x}\hat{F}$ with $\hat{F} = \hbar G \hat{a}^\dagger \hat{a}$.

recalling: $S_{FF}(\omega) = \int dt\, e^{i\omega t}\langle \hat{F}(t)\hat{F}(0)\rangle$

$$A^{\pm} = \frac{x_{ZPF}^2}{\hbar^2} S_{FF}(\omega = \mp\Omega) = g_0^2 S_{NN}(\omega = \mp\Omega)$$

the normalised spectrum is simply:

$$S_{NN}(\omega) = \int dt\, e^{i\omega t} \langle \hat{a}^\dagger \hat{a}(t)\, \hat{a}^\dagger \hat{a}(0) \rangle .$$

$$= \bar{n}_c \frac{\kappa}{\kappa^2/4 + (\Delta+\omega)^2}$$

Finally: $\bar{n}_{min} = \left(\frac{A^-}{A^+} - 1\right)^{-1} = \left(\frac{(\kappa/2)^2 + (\Delta-\Omega)^2}{(\kappa/2)^2 + (\Delta+\Omega)^2} - 1\right)^{-1}$

$\rightarrow$ $\kappa \ll \Omega$ $\Rightarrow$ $\boxed{\bar{n}_{min} = \left(\frac{\kappa}{4\Omega}\right)^2 < 1}$

$\rightarrow$ $\kappa \gg \Omega$ $\Rightarrow$ $\boxed{\bar{n}_{min} = \frac{\kappa}{4\Omega} \gg 1}$

Careful this value is for $\Delta = -\Omega$
not optimum in unresolved regime

→ Simon entanglement

→ $|\psi_{EPR}\rangle \propto |0\rangle_0 |00\rangle_{AB}$

$$+ \sqrt{p_b}\left(|H\rangle|01\rangle_{AB} + |V\rangle|10\rangle_{AB}\right)$$

$$+ \mathcal{O}(p_b)$$

→ $|\psi_{in}\rangle \propto |0\rangle + \alpha\left(\cos\frac{\theta_{in}}{2}|H\rangle + \sin\frac{\theta_{in}}{2}|V\rangle\right) + \mathcal{O}(|\alpha|^2)$

→ $|\psi_{out}\rangle = \cos\frac{\theta_{in}}{2}|10\rangle_{AB} \pm e^{i\phi_{in}}\sin\frac{\theta_{in}}{2}|01\rangle_{AB}$