Public Goods Referenda without Perfectly Correlated Prices and Quantities

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March 17, 2015

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Running Title: Public Goods Referenda

Acknowledgment:
The authors thank Trudy Cameron, Katherine Carson, Richard Carson, Koichi Kuriyama, Mike McKee, Naoko Nishimura, Jason Shogren, Christian Vossler, and participants at AERE session at the 2010 ASSA meeting, 4th WCERE, and 65th European Meeting of the Econometric Society for insightful comments and suggestions on an earlier draft of the paper. The experiment was supported by the Grant-in-Aid for Scientific Research (B) (21310030). The views expressed in the paper are those of the authors.
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Abstract:
This paper examines the incentive properties of probabilistic referenda. In contrast to earlier research in which prices and quantities are perfectly correlated, we consider uncertain and potentially different outcomes for prices and quantities. We provide a theoretical analysis on incentive compatibility and an induced-value experimental test of this theory to gain new insights. First, using a standard design, our results confirm previous findings. Second, our results suggest that moving away from a perfectly correlated design undermines the incentive compatibility result found in other studies. Third, our experimental results are consistent with choices made by risk-averse agents in our theoretical analysis. Our findings would be important for survey design in practice as well as theoretical aspect of CV referenda.

Keywords:
probabilistic referenda, incentive compatibility, contingent valuation, consequentiality, induced-values

JEL Codes:
C91 (laboratory, individual behavior), H41 (public goods), Q51 (valuation of environmental effects)
1. Introduction

Policy makers and economic analysts often want to know the public’s preferences for non-market public goods. To this end, stated-preference contingent valuation (CV) referenda have been intensively used in environmental economics and other policy related fields. Given that stated preference approaches remain the only option available for valuing many non-market public goods and that CV is the most commonly used approach, clarifying the conditions that make CV referenda incentive compatible is of first order importance. Researchers and practitioners can use survey responses to estimate true values by applying well-developed econometric methodologies if responses to CV survey questions satisfy incentive compatibility. This paper examines the conditions that make CV referenda incentive compatible by exploring the incentive properties of probabilistic referenda with cost and supply-side uncertainty.

The strategy proofness (i.e. incentive compatibility) of a binding one-shot binary referendum is a well-known theoretical result (Gibbard, 1973; Satterthwaite, 1975). However, it is generally not possible to implement “binding” CV referenda in the field. The question that matters here is whether there are conditions under which CV referenda that are not directly binding elicit truthful responses? Carson and Groves (2007) provide conditions under which a single shot, binary CV referendum with a majority decision rule is incentive compatible. The conditions provided by Carson and Groves emphasize consequentiality of the referendum’s results, i.e. the vote in the CV referendum “seen by the agent as potentially influencing an agency’s actions.” Research by Johnston (2006) that compares results from a CV referendum to an actual referendum and research by Herriges et al. (2010) that explores the impact of CV respondents’ beliefs about consequentiality suggest that indeed consequentiality matters.

Other research directly investigates the impact of consequentiality through
designed field experiments that manipulate consequentiality utilizing probabilistic referenda where binding probabilities range from 0 to 1 (Cummings and Taylor, 1998; Carson et al., 2004). Experimental evidence from these studies suggests that consequentiality of the referendum outcome is the most robust and effective means of eliminating differences between stated and actual voting behaviors (Landry and List, 2007; Vossler and Evans, 2009; Poe and Vossler, forthcoming). Cummings and Taylor (1998) conduct a laboratory experiment that employs a CV referendum for provision of a specified public good. Varying the probability that a referendum vote is binding as an experimental design, they analyze the relationship between the probability that the referendum is binding and responses to the referendum. They find that only high probabilities, those in excess of 0.5, statistically yield equal proportions in favor to that of a real referendum (i.e. the probability is 1). Carson et al. (2004) note that the results from Cummings and Taylor (1998) could be influenced by the fact that the good could be provided outside of their referendum. Carson et al. (2004) propose an alternative experiment where a unique private good is provided to each subject through a public referendum. In contrast to Cummings and Taylor (1998) they find no evidence of voting differences for non-zero binding probabilities (0<p<1) and a binding referendum (p=1). Landry and List (2007) use a similar experimental design to compare consequential referendum responses with binding referendum responses and find that consequential responses are not statistically different from real responses. The results of Carson et al. (2004) and Landry and List (2007) are consistent with the theoretical predictions suggested in Carson and Groves (2007).

Basic economic principles suggest that there are two important considerations in CV referenda: payment and provision. In the studies mentioned above that manipulate consequentiality through binding probabilities, possible economic outcomes of a
referendum are limited to only two outcomes: (1) the referendum is binding and therefore the good is provided and the payment is collected or (2) the referendum is not binding and neither the good is provided nor is the payment collected. In these experiments, provision and payment are perfectly correlated. All uncertainty is over whether the referendum is binding. However, in real world applications one could also imagine outcomes involving relatively low or high provision of the public good coupled with relatively high or low realized costs. Many projects, especially environmental, could have cost uncertainty and/or supply-side uncertainty. For example, there often exists considerable supply-side uncertainty over ecosystem restoration projects while costs are very certain. Likewise, there is considerable cost uncertainty over climate change policy to reduce greenhouse gas emissions. Costs of reduction will depend on when alternative technologies become available, the cost of those technologies, and so on. Several studies explore uncertainty over provision (Champ et al., 2002; Burghart et al., 2007; Shafran, 2007) or payment (Cameron et al., 2002; Flores and Strong, 2007) in stated preference studies, concluding these uncertainties viewed separately should influence responses to stated preference questions. Given these two potential sources of uncertainty, it follows that consequentiality should extend beyond the simple notion of whether a choice will be binding or not binding. Separating out these two dimensions of uncertainty leads us to a richer notion of consequentiality, consequentiality of a vote in provision and consequentiality of a vote in payment.

In order to capture these uncertainties in a simple model that can be used for experimental exploration, we first develop a model with a known probability of provision and a known, but separate, probability of payment. While the random variables public good (Provide or No provide) and payment (Pay or No pay) are independent in our design, they are not perfectly correlated. That is we allow for outcomes where the good is
provided at no cost or the good is not provided but subjects still pay. This expands the outcome space from two outcome probabilistic referenda (\{Pay, Provide\} \{No pay, No provide\}) to four outcome probabilistic referenda (\{Pay, Provide\}, \{No pay, No provide\}, \{Pay, No provide\}, \{No pay, Provide\}). For the purposes of comparison, we also consider the perfectly correlated, two outcome referendum used in previous studies.

We emphasize investigating the incentive compatibility of these probabilistic referenda. An important empirical question, whether the observed data can be used to estimate true value, requires considering two definitions or notions of incentive compatibility. Our first definition of incentive compatibility, and one that permeates discussions in the literature, is that a probabilistic referendum satisfies incentive compatibility if voting yes is optimal for a subject if and only if his/her valuation for the project is greater than the cost he/she pays. The second definition of incentive compatibility is that a probabilistic referendum is incentive compatible if and only if voting yes is optimal if his/her expected utility of voting yes is greater than the expected utility of voting no. Using our simple theoretical model, we show that either form of probabilistic referendum, two or four outcome, satisfies the second definition of incentive compatibility. For two outcome referenda found and discussed in the literature, the expected utility of voting yes will only exceed the expected utility of voting no if and only if the value for the project exceeds the cost paid if the referendum is binding. For our expanded four outcome referenda, we show this is not true. A risk-averse individual may vote no in the four outcome referendum though their value exceeds the cost in the case of \{Pay, Provide\}. Similarly a risk-loving individual may vote yes though their value is less than the cost of \{Pay, Provide\}. Hence attitudes toward risk matter.

In addition, we provide an induced value experimental test of our theoretical predictions. Using induced value experiments we first replicate the Carson et al. (2004)
experiments where provision and payment probabilities are the same random variable, i.e. probability the referendum is binding. Using probabilities 0, .01, .25, .75, and 1 we find incentive compatible choices for all subjects at probabilities .25, .75, and 1. Then allowing independent probabilities for provision and payment, pairs {(0, 0), (.01, .01), (.25, .25), (.5, .5), (.75, .75), (1, 1)}, we find subjects tend to vote no more often than when there is no uncertainty over provision and payment. This suggests that our experimental results are consistent with choices made by risk-averse agents in our theoretical analysis.

2. Theoretical Framework

In this paper, we explore the incentive properties of probabilistic referenda, where probabilities of the referendum being binding range from 0 to 1, without perfectly correlated prices and quantities in which there exist four potential outcomes. For the purposes of comparison, we also consider the perfectly correlated two outcome probabilistic referenda.

2.1. Setup

Let \( b \) be the cost or bids subjects pay and let \( v_i \) be subject \( i \)'s induced value (\( v_i > 0 \)).\(^1\) In a binding binary referendum with a majority vote implementation rule, if more than 50% of subjects vote yes on the proposition “contribute \( b \) to receive \( v_i \),” then the referendum has passed and therefore the payment is collected (Pay) and the good is provided (Pro). If not, the referendum has failed and neither the payment is collected (No Pay) nor is the good provided (No Pro). We can denote the outcome space of the referendum as \( \Omega_R = \)

\(^{1}\) If we assume that subject \( i \) has a linear form indirect utility function, the induced value \( v_i \) represents subject \( i \)'s Hicksian compensating surplus (variation) for the project.

\(^{2}\) We assume that there is no preference uncertainty in the sense that all subjects know their own values of the project when it is provided. We consider provision uncertainty as an independent issue from the preference uncertainty.
\{\text{Pass, Fail}\} = \{(\text{Pro, Pay}), (\text{No Pro, No Pay})\}.

Now, let us consider a probabilistic referendum (PR) with a majority vote implementation rule. The PR will be implemented by the “Two-step Referendum Rule\(^3\),” where, Step 1: If more than 50% of subjects vote yes on the proposition “contribute $b$ to receive $v_i$,” then the referendum has PASSED. If not, the referendum has FAILED. Step 2: Given the referendum passes (more than 50% of subjects vote yes), an outcome \(j\), which results in monetary payoff \(\pi_j\), occurs with probability \(p_j\). Where, \(p_j\) denotes a probability that an outcome \(j\) occurs in PR and \(\pi_j\) denotes a monetary payoff when an outcome \(j\) occurs.

By identifying the outcome space and subject’s payoff in each referendum, we define the perfectly correlated two outcome probabilistic referenda (TOPR) and the not perfectly correlated four outcome probabilistic referenda (FOPR). In Step 1 of PR, if the referendum fails, then the outcome and subject’s payoff are the same as those for fail in binding binary referenda. That is, the outcome given the referendum fails is that neither the good is provided nor is the payment collected: (No Pro, No Pay). Now, let \(y\) denote income or initial endowment \((y > b)\).\(^4\) Then, all subjects receive their initial endowment \(y\). On the other hand, if the referendum passes in Step 1, there is a probabilistic nature with respect to payment and provision in Step 2 of the PR. The TOPR has two possible outcomes: (1) the referendum is binding, which occurs with probability \(p\), and therefore the good is provided and the payment is collected or (2) the referendum is not binding, which occurs with probability \((1 - p)\), and neither the good is provided nor is the payment collected. The probabilistic outcomes in Step 2 of the TOPR are given by \(j \in \Omega_{\text{TOPR|Pass}} =\)

\(^3\) Cumming and Taylor (1998) and Carson et al. (2004) also identify PR using a two-step rule.

\(^4\) Though we assume homogeneous income distributions \((y)\) and homogeneous costs \((b)\) for all subjects, these assumptions are not essential and our results regarding the incentive properties do not change.
{(Pro, Pay), (No Pro, No Pay)}. The first outcome (Pro, Pay) results in the payoff of \( y + v_i - b \). The second outcome (No Pro, No Pay) results in the payoff of \( y \).

The probabilistic outcomes in Step 2 of our FOPR design are given by \( j \in \Omega_{\text{FOPR|Pass}} = \{(Pro, Pay), (No Pro, Pay), (Pro, No Pay), (No Pro, No Pay)\} \). That is, the FOPR provides two outcomes in addition to those in the TOPR. While the probabilities of payment and provision are the same in this paper, the outcomes are two distinct and independent random variables.\(^5\) Let \( p \) be the probability that the cost is collected and let the same \( p \) be the probability that the good is provided. The first outcome (Pro, Pay) occurs with probability \( p^2 \) and results in the payoff of \( y + v_i - b \). The second outcome (No Pro, Pay) occurs with probability \((1 - p) \cdot p\) and results in the payoff of \( y - b \). The third outcome (Pro, No Pay) occurs with probability \((1 - p) \cdot p\) and results in the payoff of \( y + v_i \). The last outcome (No Pro, No Pay) occurs with probability \((1 - p)^2\) and results in the payoff of \( y \).

2.2. Voting Decisions

Let \( \eta(d_i, D_{-i}) \) represent voter \( i \)'s subjective probability of passing\(^6\), where \( d_i \in \{\text{Yes, No}\} = \{1, 0\} \) is voter \( i \)'s decision, and \( D_{-i} = (d_1, d_2, \ldots, d_{i-1}, d_{i+1}, \ldots, d_N) \) is the vector containing the decisions of the other \( N - 1 \) subjects.\(^7\) By the definition of majority rule, we can identify \( \eta(d_i, D_{-i}) \) as follows:

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\(^5\) Mitani and Flores (2010) deal with the case in which the probability of payment is not equal to the probability of provision, using a threshold provision mechanism. Their experimental analysis suggests that the relative importance between payment and provision uncertainty plays an important role for the explanation of hypothetical bias.

\(^6\) This probability is different and independent from the exogenous probability that the referendum is binding or not (\( p \)).

\(^7\) That is, the message space is binary.
\[ \eta(d_i, D_{-i}) = \Pr \left[ \frac{\sum_{k=N}^{d_i} d_k}{N} > 0.5 \right] = \Pr \left[ \frac{(d_i + D_{-i})' I_{N-1}}{N} > 0.5 \right], \]  

(1)

where \( I_{N-1} \) denotes an \((N - 1, 1)\) unit vector. Consistent with field applications, we assume that subjects know only their own values. In other words, they do not have any information about the distribution, i.e. neither the range nor the frequency of values. Thus, for subject \( i \), \( D_i \) is unknown while \( d_i \) is his/her own decision. We can treat the scalar \( D_i' \cdot I_{N-1} \) as a random variable. The variable \( D_i' \cdot I_{N-1} \) takes an integer value from the range \([0, N-1]\).\(^8\) Note that there exists a value of \( D_i' \cdot I_{N-1} \) for which subject \( i \) becomes pivotal, in the sense that subject \( i \)'s vote is decisive in breaking a tie. Let \( D_i^* \) be the others' decision vector such that subject \( i \) is pivotal, then \( D_i = \{D_i^* \text{ or } \neg D_i^*\} \). If subject \( i \) is pivotal, the following holds for \( D_i^* \):

\[ \eta(1, D_i^*) > \eta(0, D_i^*), \text{ for } D_i^* \text{ (that is, } \eta_Y - \eta_N > 0). \]  

(2)

This implies that subject \( i \)'s subjective probability of passing when he/she votes yes is greater than that of passing when he/she votes no, if he/she is pivotal. Next, if subject \( i \) is not pivotal, the following holds for all \( \neg D_i^* \):

\[ \eta(1, \neg D_i^*) = \eta(0, \neg D_i^*), \text{ for all } \neg D_i^* \text{ (that is, } \eta_Y - \eta_N = 0). \]  

(3)

Combining these two statements, we have the following lemma:

**Lemma 1** (Subjective Probability of Passing) For all \( D_i \), \( \eta_Y - \eta_N = \eta(1, D_i) - \eta(0, D_i) \geq 0. \)

For at least one \( D_i^* \), \( \eta_Y - \eta_N = \eta(1, D_i^*) - \eta(0, D_i^*) > 0. \)

Let us assume that the subject has an increasing utility function of the monetary payoff: \( U(\pi) \). Now, we define the expected utility given a referendum result. First, the

\[^8\text{ Let } \sigma_i \text{ be a random variable } D_i \cdot I_{N-1}, \text{ which takes an integer value } \{0, 1, 2, \ldots, N-1\} \text{ with probability vector } \rho_i = (\rho_0, \rho_1, \rho_2, \ldots, \rho_{N-1})', \text{ where } \rho_j = \Pr[\sigma_i = j] \text{ and } \sum_{k=0}^{N-1} \rho_k = 1. \text{ Then, } \eta(d_i, D_i) = 1 - \Pr[0.5N - d_i > \sigma_i] = 1 - \sum_{k=0}^{0.5N-d_i} \rho_k. \text{ Note that subjects have no idea of the distribution of } \rho_i \text{ due to incomplete information about others' preferences and no communication among subjects.} \]**
expected utility given the referendum passes (PASS; $P$) is defined as follows:

$$EU_P = \sum_{j \in \Omega} p_j U(\pi_j).$$  \hspace{1cm} (4)

Second, the expected utility given the referendum fails (NOT PASS; $NP$) is defined as follows:

$$EU_{NP} = U(0).$$  \hspace{1cm} (5)

Then, we can define the expected utility of voting yes and no, using $EU_P$ and $EU_{NP}$. The expected utility voting yes is given as follows:

$$EU_Y = \eta(1, D, i) EU_P + \left( 1 - \eta(1, D, i) \right) EU_{NP}. \hspace{1cm} (6)$$

Likewise, the expected utility of voting NO is given as follows:

$$EU_N = \eta(0, D, i) EU_P + \left( 1 - \eta(0, D, i) \right) EU_{NP}. \hspace{1cm} (7)$$

Now, consider the expected utility difference between voting yes and no. By the definition of the expected utility of voting yes and no, we have the following:

$$EU_Y - EU_N = \left( \eta_Y - \eta_N \right) \left( EU_P - EU_{NP} \right). \hspace{1cm} (8)$$

This equation implies that if $\eta_Y - \eta_N > 0$, that is if subject $i$ is pivotal or at least the probability that subject $i$ is pivotal is positive\textsuperscript{10}, then in a dichotomous choice referenda with majority-vote rule the sign of the expected utility difference between voting yes and no depends only on the sign of the difference between the expected utility given the

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\textsuperscript{9} Without loss of generality, we set $y = 0$ hereafter in this section.

\textsuperscript{10} As an alternative model, we consider a model where voters form a subjective probability that they are pivotal, similar to the model that Vossler and Evans (2009) employ. Let $EU_{di}$ be voter $i$’s expected utility from voting decision $d_i = \{ \text{YES, NO} \}$. Let $\eta_i^P, \eta_i^{NP:PASS}$, and $\eta_i^{NP:FAIL}$ be the probability that voter $i$ is pivotal, the probability that voter $i$ is not pivotal and the referendum passes, and the probability that voter $i$ is not pivotal and the referendum fails, respectively. Now, voter $i$'s expected utility from voting decision is given by

$$EU_{di} = \eta_i^P \left( d_i EU_P + \left( 1 - d_i \right) EU_{NP} \right) + \eta_i^{NP:PASS} EU_P + \eta_i^{NP:FAIL} EU_{NP}. \hspace{1cm} (9)$$

Then, we have the following:

$$EU_Y - EU_N = \eta_i^P \left( EU_P - EU_{NP} \right). \hspace{1cm} (10)$$

Combing equations (8) and (10), we see that $\eta_Y - \eta_N > 0$ implies $\eta_i^P > 0$.\textsuperscript{11}
referendum passes and the expected utility given the referendum fails. We have two probabilities in our model: (1) a subjective probability of passing ($\eta$) given subject’s voting decision and (2) exogenous probability that the referendum is binding or not ($p$). Note that we assume that these two probabilities are independent. In our theoretical model and experimental test, this assumption is not restrictive at all because we employ the two step voting rule, which allows us to deal with these two probabilities as independent.

2.3. Incentive Properties of Probabilistic Referenda

A voting mechanism for eliciting individual preferences is said to be incentive compatible when the individual’s optimal voting decision (e.g. dominant strategy) is to truthfully reveal their preferences. We define the incentive compatibility of probabilistic referenda as follows. First, we consider a situation without uncertainty. A (probabilistic) referendum is incentive compatible (IC1) if voting yes is optimal for subject $i$ if and only if his/her valuation for the project is greater than the cost he/she pays:

$$\text{IC1: } v_i > b_i \iff \text{subject } i \text{ votes yes.}$$

Then, we need to consider a situation where there exists uncertainty, because of a probabilistic nature in Step 2 of PR. In such a situation with uncertainty, a probabilistic referendum is incentive compatible (IC2) if voting yes is optimal for subject $i$ if and only if the expected utility from voting yes is greater than the expected utility from voting no:

$$\text{IC2: } EU_Y > EU_N \iff \text{subject } i \text{ votes yes.}$$

It is ideal if probabilistic referenda satisfy the first incentive compatibility (IC1) because not only the referendum mechanism guarantees that the referendum result is based on truth revelations but also researchers or policy makers can estimate the distribution of true values using only information about voting decisions ($d_i$) and the costs ($b_i$). The probabilistic referenda mechanisms that satisfy only the second incentive compatibility (IC2) are still truth revealing in the sense that no subject has an incentive to lie to reveal
his/her preference. However, from the perspective of researchers or policy makers, it is almost impossible to estimate the distribution of true values using only available information for them, voting decisions and the costs, without additional information. We show here that there exist cases that probabilistic referenda satisfy the second incentive compatibility (IC2) but not the first one (IC1), and that subject’s risk attitude plays an important role for the alignment of IC1 and IC2. In this case, to estimate the distribution of true values, we would need additional information about subject’s individual risk attitude.\footnote{\small{Though we treat only the case that the probability of payment (payment uncertainty) is equal to the probability of provision (provision uncertainty) in this paper, if these probabilities are different from each other then we would need to measure these probabilities in addition to subject’s risk attitude. In other words, satisfying the first incentive compatibility (IC1) allows us to ignore the influence of the probabilistic nature and subject’s risk attitude toward the uncertainty. This implies that it is important for probabilistic referenda mechanisms to satisfy the first incentive compatibility (IC1) from the perspective of eliciting individual preferences for public projects.}}

Hereafter in this section, we explore the incentive properties of the TOPR and the FOPR. First, we have a convenient result to use in examining the incentive compatibility of PR, which all \textit{binary} probabilistic referenda would satisfy. Using equation (8) and Lemma 1, if $EU_P - EU_{NP} > 0$, the following holds:

\begin{align}
EU_Y (D_i) &\geq EU_N (D_i), \text{ for all } D_i, \text{ and } \quad (11) \\
EU_Y (D^*_i) &> EU_N (D^*_i), \text{ for at least one } D^*_i. \quad (12)
\end{align}

Thus, we have the following lemma.

\textbf{Lemma 2 (Weakly Dominant Strategy in any Binary PR)} \textit{Voting yes is a weakly dominant strategy, if and only if $EU_P - EU_{NP} > 0$.}

This lemma allows us to focus on investigating whether the inequality $EU_P - EU_{NP} > 0$ holds or not, when exploring the incentive properties of PR.
**Proposition 1** (IC of Referenda) *A binding binary referendum is incentive compatible, in the sense that a binding binary referendum satisfies the first incentive compatibility (IC1).*

**Proof:** By the definition of the payoff in CV referenda, \( EU_P - EU_{NP} = U(v_i - b) - U(0) \). Since the utility function is increasing in the monetary payoff, \( EU_P > EU_N \) if and only if \( v_i - b > 0 \). Thus, by Lemma 2, for subject \( i \), voting yes is a weakly dominant strategy if and only if \( v_i - b > 0 \). This satisfies the IC1. □

This result is completely consistent with the traditional well-known theorem of Gibbard (1973) and Satterthwaite (1975), which is the starting point in the Carson and Groves (2007) theoretical framework. Also, this is consistent with the result on strategic behavior in CV referenda by Hoehn and Randall (1987).

**Proposition 2** (IC of the TOPR) *Suppose that the referendum is consequential (i.e. \( p > 0 \)), then the two outcome probabilistic referendum (TOPR) is incentive compatible, in the sense that the TOPR satisfies the first incentive compatibility (IC1).*

**Proof:** Let \( F_{NP}(\pi \mid 1>p>0, v, b) \) be a cumulative distribution function of the monetary payoff given the referendum fails (NP: Not Pass). Let \( F_{P,T}(\pi \mid 1>p>0, v, b) \) be a cumulative distribution function of the random monetary payoff given the referendum passes in the TOPR. For \( v - b > 0 \), since \( F_{NP}(\pi) \geq F_{P,T}(\pi) \) for all \( \pi \), \( F_{P,T}(\pi) \) first-degree stochastically dominates (FSD) \( F_{NP}(\pi) \). This implies that \( \int U(\pi) F_{P,T}(\pi) d\pi \geq \int U(\pi) F_{NP}(\pi) d\pi \) (i.e. \( EU_{P,T} \geq EU_{NP} \)) for all increasing \( U(\pi) \), as long as \( v - b > 0 \). Thus, by Lemma 2, for \( p > 0 \), voting yes is a weakly dominant strategy if and only if \( v - b > 0 \). This satisfies the IC1. □

This result confirms Carson and Groves (2007) who mention that the probability that a CV referendum is consequential does not influence its incentive properties as long

\[ EU_{P,T} = p U(v - b) + (1 - p) U(0). \]
as the probability is positive, implying that consequential probabilistic referenda are incentive compatible. The result also supports a growing body of recent experimental evidence that consequential treatments from probabilistic referenda provide outcomes that are statistically indistinguishable from outcomes of binding non-probabilistic referenda (Carson et al., 2004; Landry and List, 2007; Vossler and Evans, 2009). Now, we show our main results about the incentive compatibility of the FOPR. We find the result on the incentive compatibility of the FOPR depends on a subject’s risk attitude. Note that while we refer to a scalar \( p \) below, we are referring to two separate, but equal probabilities, the probability of payment and the probability of provision. We use a scalar notation because in this paper we are only considering provision and payment probabilities that are equal, though the random variables are separate and independent.

**Proposition 3-1** (IC of the FOPR: Risk-neutral Agents) For \( p > 0 \), if the utility function is linear (i.e. risk-neutral agents), the four outcome probabilistic referendum (FOPR) is incentive compatible, in the sense that the FOPR satisfies the first incentive compatibility (IC1).\(^{13}\)

**Proposition 3-2** (IC of the FOPR: Risk-averse/lover Agents) For \( 1 > p > 0 \), if the utility function is concave (i.e. risk-averse agents) or convex (i.e. risk-loving agents), the FOPR is NOT incentive compatible, in the sense that the FOPR does not satisfy the first incentive compatibility (IC1) whereas the FOPR satisfies the second incentive compatibility (IC2).

**Proof:** Following the Proposition 2, let \( F_{p,F}(\pi \mid 1>p>0, \nu, b) \) be a cumulative distribution function of the random monetary payoff given the referendum passes in the FOPR. For any \( \nu - b \), since \( F_{p,F}(\pi) \) is a mean-preserving spread of \( F_{p,T}(\pi), F_{p,F}(\pi) \)

\(^{13}\) This is still true if the utility function is quasi-linear such as \( U(\nu - b) = U(\nu) - b \). However, we do not need to consider the possibility that the utility function is quasi-linear in this paper because we measure both \( \nu \) and \( b \) in the same monetary terms.
second-degree stochastically dominates (SSD) \( F_{P,F}(\pi) \). This is equivalent with the statement that \( \int U(\pi) \ F_{P,T}(\pi) \ d\pi \geq \int U(\pi) \ F_{P,F}(\pi) \ d\pi \) (i.e. \( EU_p^T \geq EU_p^F \))\(^{14}\) for all increasing and concave \( U(\pi) \) (Hadar and Russell, 1969). This also implies that \( EU_p^F \geq EU_p^T \) for all increasing and convex \( U(\pi) \). Since linear functions of the form \( U(\pi)=a\pi+b \) are both concave and convex, \( EU_p^T \geq EU_p^F \) and \( EU_p^F \geq EU_p^T \), implying that \( EU_p^T = EU_p^F \) for all increasing and linear \( U(\pi) \). Combined with Proposition 2, if the utility function is linear for \( p > 0 \), the FOPR satisfies the IC1. On the other hand, if the utility function is increasing and concave (convex) for \( 1 > p > 0 \), there exists \( \nu > b \) (\( \nu < b \)) such that \( EU_{NP} > EU_p^F \) (\( EU_{NP} < EU_p^F \)). This implies that if the utility function is concave or convex, the FOPR does not satisfy the IC1. \( \square \)

Proposition 3 suggests that a subject’s risk attitude matters when considering the incentive properties of the FOPR. Proposition 3 implies that it is possible that risk-averse subjects are likely to vote no even if their valuations are slightly greater than the costs. Likewise, Proposition 3 implies that risk-loving subjects are likely to vote yes even if their valuations are slightly less than the costs. Thus, in the FOPR, subjects whose values are close to the bids (costs) possibly fail to vote their true preferences in the sense of the first incentive compatibility (IC1). In other words, when the value is close to the bid (cost), the first incentive compatibility (IC1) of the FOPR is likely to be violated. These results suggest that researchers or policy makers who utilize only information about voting decisions and the costs possibly underestimate or overestimate their true values.

2.4. Experimental Treatment Effect

Now, we provide the theoretical implications of the treatment effects of the FOPR

\(^{14}\) \( EU_p^F \) denotes the expected utility given the referendum passes in the FOPR. Using equation (4),

\[
EU_p^F = p^2 U(v - b) - p(1 - p)[U(v) + U(-b)] + (1 - p^2) U(0).
\]
compared to the TOPR. First, we define the *treatment effects* ($\gamma$) of the FOPR compared to the TOPR as follows:

$$\gamma = EU_P^F - EU_P^T,$$  \hspace{1cm} (15)

where $EU_P^T$ and $EU_P^F$ represent the expected utility given the referendum passes in the TOPR and the FOPR, respectively. The superscript $T$ denotes the TOPR and the superscript $F$ denotes the FOPR. We omit the superscript for the expected utility given the referendum fails because the expected utility given the referendum fails in the TOPR is equal to that in the FOPR. That is, $EU_{NP} = EU_{NP}^T = EU_{NP}^F$.

Consider probit (logit) estimation using experimental data in which the dependent variable is voting yes. By Lemma 2, the probability of voting yes in the estimation is given by:

$$\Pr \left[ \text{Vote for YES} \right] = \Pr \left[ EU_P - EU_{NP} > 0 \right]. \hspace{1cm} (16)$$

This implies that the difference $EU_P - EU_{NP}$ has a positive effect on the probability of voting yes. Let $D^{FOPR}$ be a treatment dummy variable equaling 1 if the experimental design is the FOPR, 0 otherwise. Now, using this dummy variable, we can rewrite the difference as follows:

$$EU_P - EU_{NP} = EU_P^T - EU_{NP} + D^{FOPR} (EU_P^F - EU_P^T).$$ \hspace{1cm} (17)

We can see the treatment effect (15) as a coefficient of the treatment dummy variable ($D^{FOPR}$). Combining with a probit (logit) estimation-probability (16), we see that the treatment effect ($\gamma$) can be estimated using experimental data. Also, equation (17) implies that the treatment effect has a positive effect on the likelihood of voting yes. Now, we achieve the following theoretical predictions.

**Proposition 4** (Treatment Effect and Risk Attitude) *Suppose $1 > p > 0$, then the relationship between the sign of the estimable treatment effect and the utility function form are given as follows: I) Positive treatment effect of the FOPR ($\gamma > 0$) implies that the*
utility function $U$ is convex, which is consistent with that subjects are Risk-lover agents; 2) Negative treatment effect ($\gamma < 0$) implies that the utility function $U$ is concave, which is consistent with that subjects are Risk-averse agents; 3) No treatment effect ($\gamma = 0$) implies that the utility function $U$ could be linear\(^{15}\), which is consistent with that subjects are Risk-neutral agents.

**Proof:** Following the Proposition 3, $F_{p,T}(\pi)$ SSD $F_{p,F}(\pi)$. This implies the following: 1) if the utility function is increasing and linear, then $EU_p^F = EU_p^T$, implying $\gamma = 0$ (no treatment effect of the FOPR); 2) if the utility function is increasing and convex, then $EU_p^F \geq EU_p^T$, implying $\gamma \geq 0$ (non-negative treatment effect); 3) if the utility function is increasing and concave, then $EU_p^F \leq EU_p^T$, implying $\gamma \leq 0$ (non-positive treatment effect). □

Regarding the first incentive compatibility (IC1) of the FOPR experimental design, once we admit that the TOPR is incentive compatible in the sense that the TOPR satisfies the IC1, Proposition 4 implies that 1) positive treatment effect suggests that the FOPR design is not incentive compatible and interpretable as overstating subjects’ values when comparing it to the results from the TOPR design; 2) negative treatment effect suggests that the FOPR design is not incentive compatible and interpretable as understating subjects’ values when comparing it to the results from the TOPR design; 3) no treatment effect suggests that the FOPR design is incentive compatible in the sense that the experimental data from the FOPR produces the same results as that from the TOPR.

As mentioned above, we can estimate the treatment effect $\gamma$ by using our experimental data.

\(^{15}\) If the utility function is strictly increasing and continuously differentiable, no treatment effect holds if and only if the utility function is linear.
3. Experimental Design

The experiments were designed to provide a strict test of our theoretical predictions about the incentive properties of probabilistic referenda. In particular, we focus on examining the treatment effect of the FOPR design compared to the TOPR design. Varying the probability that the referendum is binding as additional experimental treatments in each design, we investigate how subjects’ voting decisions change with the consequentiality of referenda.

We employed a between-subjects one-shot induced-value experimental design, which is completely consistent with our theoretical framework. Twelve experimental sessions were conducted with two hundred and thirty subjects at the laboratory for political economy at Waseda University. Subjects were recruited from the general population at the university. The number of subjects in each experimental session is shown in Table 1. Subjects in groups of five were initially endowed with 1000 JPY (10 USD at the exchange rate of 100 JPY = 1.00 USD). Subjects were assigned to a lab computer with privacy shields; communication was not allowed between subjects. That is, during the experiment, subjects were visually isolated. The Z-tree software (Zurich Toolbox for Readymade Economic Experiments) was used in our experiments. Fifteen to thirty-five subjects in a session were randomly and anonymously assigned into three to seven groups of five subjects for each of the twelve experimental sessions. These controls for anonymity are important to minimize the possible impacts of social networks on the voting decision (List et al., 2004). Likewise, our experimental design attempts to avoid introducing potential biases due to uncertain values, group size, or other-regarding preferences (Vossler and McKee, 2006; Bohara et al., 1998; Burton et al., 2007).

*Induced-values*

Before starting the referendum vote, subjects were presented their induced-value in JPY
using a value card, which indicates each subject’s value for the project if provided. Induced-values across the session members were uniformly distributed over the range of 100JPY (1USD) to 900JPY (9USD), in 200JPY (2USD) increments (i.e. 100, 300, 500, 700, 900JPY). Experimental instructions were developed parallel to the work of Vossler and McKee (2006) and Taylor et al. (2001). Consistent with field applications, subjects were told that values varied across subjects but were not told the range and the frequency of values. That is, subjects knew only their own values.

Referendum Rules

The instruction and procedure of our experiments including the description of the probabilistic natures in the referenda followed the two-step referendum rules of Carson et al. (2004) and Cummings and Taylor (1998). The homegrown value setting that the previous studies employed was modified into the induced value setting. We further extended their perfectly correlated design (TOPR) to the four outcome not perfectly correlated design (FOPR). The cost, which subjects have to pay if the referendum passes and is binding, and the probability, that the referendum is binding, were common knowledge in each session. The Two-Step Referendum Rule (TOPR Version) is described as follows:

Two-Step Referendum Rule (TOPR Version)

**Step 1:** If more than 50% of you vote YES on this proposition (“would you pay the cost 400JPY to provide the project which gives you the value of $v_i$ JPY”), then the referendum has passed. If the referendum passes, then in Step 2 we will determine if the referendum is binding, depending on the pre-announced probability $[p]$ (assigned from \{0, 0.01, 0.25, 0.5, 0.75, 1\}). If the referendum does not pass, then no one will pay 400JPY or receive your value of the project. Everyone receives 1000JPY.

**Step 2:** Given the referendum passes, the computer determines whether the
referendum result done in Step 1 will be binding depending on the pre-announced probability \([p]\). If the referendum is binding, all of you have to pay the cost of 400JPY and you can receive your value of the project \([v_i]\)JPY.

Also, the Two-Step Referendum Rule (FOPR Version) is described as follows:

**Two-Step Referendum Rule (FOPR Version)**

**Step 1:** The same as Step 1 in the TOPR.

**Step 2:** Given the referendum passes, the computer determines whether the cost 400JPY and the provision of the project will be respectively (FOPR) binding depending on the pre-announced probabilities \([p, p]\). The chance of coercive collection is \([100p]%\), and the chance of secure provision is \([100p]%\). Your earning will depend both on whether you have to pay the cost of 400JPY and on whether you can receive your value of the project \([v_i]\)JPY.

The two-step referendum rules allow us to deal with the probability that the referendum is binding \(p\) as independent events from the subjective probability that the referendum passes \(\eta_y\).

**Experimental Treatments**

The twelve treatments and the number of subjects in each treatment are shown in Table 1. Following the experimental design of early work by Cummings and Taylor (1998) we used the same five probability values, 0, 0.25, 0.5, 0.75, 1, plus a very small probability, 0.01. There were three to seven groups of five subjects for each of the twelve experimental treatments.

**Procedures**

The experimenter provided oral instructions with a front screen and answered any questions. The instructions contained information about a vote for a public project, induced values, uncertainty over the referendum (in the TOPR treatments), uncertainty
over payment and provision (in the FOPR treatments), and the two-step referendum rule, in this order. In this one shot referendum, the probability that the referendum will be binding was publicly announced; then the subjects made their voting decisions. The experiment concluded with a questionnaire that collected basic demographics including gender and age.

Table 1: Treatments and the Number of Subjects

<table>
<thead>
<tr>
<th>TOPR (Two Outcomes)</th>
<th>FOPR (Four Outcomes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td># Subjects</td>
</tr>
<tr>
<td>$p = 0.00$</td>
<td>15</td>
</tr>
<tr>
<td>$p = 0.01$</td>
<td>35</td>
</tr>
<tr>
<td>$p = 0.25$</td>
<td>20</td>
</tr>
<tr>
<td>$p = 0.50$</td>
<td>15</td>
</tr>
<tr>
<td>$p = 0.75$</td>
<td>20</td>
</tr>
<tr>
<td>$p = 1.00$</td>
<td>20</td>
</tr>
<tr>
<td><strong>Total #</strong></td>
<td><strong>125</strong></td>
</tr>
</tbody>
</table>

4. Results

The experiments here investigate the treatment effect of the FOPR compared to the TOPR. We begin our analysis with a quick look at the general patterns at the aggregate level. The individual level econometric analysis is then used to test our fundamental theoretical predictions about the treatment effect.

Figure 1 shows the observed percentage of subjects voting yes in each treatment. Also, Table 2 reports the overall voting results. In the table, the $v$ denotes induced values and $b$ denotes the cost. For example, the number shown in the column of Vote for YES and $v > b$ (that is, the third column from the left) represents the number of subjects voting yes whose values are greater than the cost, implying that these voting decisions satisfy the first incentive compatibility (IC1). In our induced value design, 60% of subjects should
vote yes in this one shot referendum. For probabilities 0.25 or greater, we see that 60\% of subjects voted yes for the TOPR design but not for our alternative FOPR design (see Figure 1). The last column from the left in Table 2 shows the percentage of subjects’ voting decisions satisfying the first incentive compatibility (IC1) at each treatment. For the FOPR design, we can see some observations in the sixth column of Vote for NO and \(v > b\), implying understatements of their values in the sense that the observations violate the first incentive compatibility (IC1). We should note that while the observations that fall into the fourth (looks like overstatement) and sixth (looks like understatement) column in Table 2 violate the first incentive compatibility (IC1), for subjects in the FOPR the observations might still be consistent with truth revealing, in the sense of the second incentive compatibility (IC2).

![Figure 1: Summary of Results](image-url)
Table 2: Voting Results

<table>
<thead>
<tr>
<th>Treatments</th>
<th>Vote for YES</th>
<th></th>
<th>Vote for NO</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$v &gt; b$</td>
<td>$v &lt; b$</td>
<td>Total</td>
<td>$v &gt; b$</td>
</tr>
<tr>
<td>TOPR 0</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>TOPR 0.01</td>
<td>15</td>
<td>0</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>TOPR 0.25</td>
<td>12</td>
<td>0</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>TOPR 0.5</td>
<td>9</td>
<td>0</td>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>TOPR 0.75</td>
<td>12</td>
<td>0</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>TOPR 1</td>
<td>12</td>
<td>0</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>65</td>
<td>0</td>
<td>65</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>46</td>
<td>0</td>
<td>46</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 3 presents the probit estimation results in which the dependent variable is vote for yes. The explanatory variables include: 1) the treatment effect dummy of the FOPR ($D_{FOPR}$) which is equal to 1 if the vote is made in the FOPR treatments and also the vote is made in the probability is greater than 0 and less than 1 (i.e. $0 < p < 1$); 2) the treatment effect dummies of the probabilities that the referendum is binding ($D_{p=0.01}$, $D_{p=0.25}$, $D_{p=0.5}$, $D_{p=0.75}$), which are compared to the treatment in which the probability is 1 (i.e. binding binary referenda); 3) subject’s induced value ($v$); 4) gender ($Gender$) which is equal to 1 if subject’s gender is male; 5) age ($Age$); and 6) the dummy variable ($Econ$) which is equal to 1 if the subject is an economics student. Here is our main concern. The treatment effect coefficient ($D_{FOPR}$) is negative and statistically significant at the 5% level in both models with (Model 1) and without (Model 2) demographic variables. From Proposition 4 in section 2, this implies that the result is consistent with choices made by risk-averse agents in our theoretical analysis. The estimates of treatment effects of
probabilities of being binding suggest that voting results observed for probabilities 0.25 or greater but less than 1 are statistically no different from those observed in a binding binary referendum. This result is consistent with previous findings from studies that employed homegrown-value probabilistic referenda to examine consequentiality (Carson et al., 2004; Landry and List, 2007). The coefficient of induced values is positive and significant at the 1% level. So, subjects who have higher values are more likely to vote for yes. Also, we did not find any individual effects like gender, age, or economics major. This result regarding gender is different from the results reported by Mitani and Flores (2010) where the not incentive compatible provision point mechanism is employed.

Table 3: Probit Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th></th>
<th></th>
<th>Model 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ests</td>
<td>S.E.</td>
<td>MargEff</td>
<td>Ests</td>
<td>S.E.</td>
<td>MargEff</td>
</tr>
<tr>
<td>Const.</td>
<td>-1.657</td>
<td>1.243</td>
<td>-0.654</td>
<td>-1.919</td>
<td>0.373 ***</td>
<td>-0.759</td>
</tr>
<tr>
<td>$D_{FOPR}^{p=0}$</td>
<td>-0.725</td>
<td>0.285 **</td>
<td>-0.274</td>
<td>-0.654</td>
<td>0.277 **</td>
<td>-0.249</td>
</tr>
<tr>
<td>$D_{p=0.01}$</td>
<td>-1.362</td>
<td>0.444 ***</td>
<td>-0.422</td>
<td>-1.357</td>
<td>0.429 ***</td>
<td>-0.426</td>
</tr>
<tr>
<td>$D_{p=0.25}$</td>
<td>-0.803</td>
<td>0.404 **</td>
<td>-0.292</td>
<td>-0.771</td>
<td>0.394 **</td>
<td>-0.284</td>
</tr>
<tr>
<td>$D_{p=0.5}$</td>
<td>-0.155</td>
<td>0.445</td>
<td>-0.06</td>
<td>-0.209</td>
<td>0.431</td>
<td>-0.082</td>
</tr>
<tr>
<td>$D_{p=0.75}$</td>
<td>-0.056</td>
<td>0.433</td>
<td>-0.022</td>
<td>-0.090</td>
<td>0.431</td>
<td>-0.035</td>
</tr>
<tr>
<td>$v$</td>
<td>0.005</td>
<td>0.001 ***</td>
<td>0.002</td>
<td>0.002</td>
<td>0.001 ***</td>
<td>0.002</td>
</tr>
<tr>
<td>Gender</td>
<td>-0.279</td>
<td>0.251</td>
<td>-0.11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.013</td>
<td>0.057</td>
<td>-0.518</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Econ</td>
<td>0.438</td>
<td>0.307</td>
<td>0.173</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>230</td>
<td></td>
<td></td>
<td>230</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LogL</td>
<td>-81.554</td>
<td></td>
<td></td>
<td>-83.255</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LRI</td>
<td>0.488</td>
<td></td>
<td></td>
<td>0.477</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The *** and ** denote parameter is statistically different from zero at the 1% and 5% significance levels. MargEff denotes “Marginal Effect.” LRI denotes “Likelihood Ratio Index.”

Our results in the standard TOPR confirm previous findings of PR (Carson and Groves, 2007; Carson et al., 2004). Since all previous studies (Landry and List, 2007; Carson et al., 2004; Cumming and Taylor, 1998) of PR employed homegrown values, our
study is the first induced-value test of PR. The results of the FOPR suggest that moving away from perfectly correlated prices and quantities undermines the incentive compatibility result found in other studies. The experimental results are consistent with choices made by risk-averse agents in our theoretical analysis.

We here provide a closer look at the effect of risk-aversion on voting behavior at the individual level. First of all, the fourth and seventh columns in Table 2 show that for subjects whose values are less than the cost in the FOPR treatments of $0.01 < p < 1$, all votes meet the first incentive compatibility (IC1), implying that no subjects violate the IC1 because of risk-loving. On the other hand, the third and sixth columns in Table 2 show that $38\%$ of subjects whose values are greater than the cost in the FOPR treatments of $0.01 < p < 1$ vote no, implying that these votes violate the IC1. The question we discuss here is how risk-averse would a subject need to be for them to vote no even though their value exceeds the cost?\textsuperscript{16}

To calculate the degree of relative risk aversion required for subjects whose values are greater than the cost to vote no, we assume the standard constant relative risk-aversion utility function:

$$U(\pi) = \pi^{1-\rho} / (1 - \rho), \text{ for } \rho \neq 1 \text{ and } U(\pi) = \ln \pi, \text{ for } \rho = 1,$$

where $\rho = -\pi \ U''(\pi) / U'(\pi)$ is the coefficient of relative risk aversion. $\rho = 0$ implies risk neutrality, whereas $\rho > 0$ implies risk aversion. Under this specification, we numerically calculate the coefficients of relative risk aversion such that $EU_P(p, v, b) - EU_{NP}(p, v, b) = 0$, for $0.01 < p < 1$ and $v > b$, where $EU_P$ and $EU_{NP}$ respectively denote the expected utility given the referendum passes and the expected utility given the referendum fails. The calculated coefficients under this constraint would be a threshold that changes

\textsuperscript{16} We consider the expected utility maximizers. Another possible explanation would be loss aversion since observed bias in the experiments is one direction, i.e. voting no in the FOPR.
theoretically expected voting decisions from yes to no.

Table 4: Observations and the Coefficients of Relative Risk-aversion

<table>
<thead>
<tr>
<th>p</th>
<th>v-b</th>
<th># vote Yes</th>
<th># vote No</th>
<th>% vote No</th>
<th>CRRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>100</td>
<td>1</td>
<td>3</td>
<td>75%</td>
<td>0.7</td>
</tr>
<tr>
<td>0.25</td>
<td>300</td>
<td>4</td>
<td>0</td>
<td>0%</td>
<td>1.4</td>
</tr>
<tr>
<td>0.25</td>
<td>500</td>
<td>3</td>
<td>1</td>
<td>25%</td>
<td>1.8</td>
</tr>
<tr>
<td>0.5</td>
<td>100</td>
<td>2</td>
<td>2</td>
<td>50%</td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>300</td>
<td>3</td>
<td>1</td>
<td>25%</td>
<td>1.9</td>
</tr>
<tr>
<td>0.5</td>
<td>500</td>
<td>4</td>
<td>0</td>
<td>0%</td>
<td>2.3</td>
</tr>
<tr>
<td>0.75</td>
<td>100</td>
<td>2</td>
<td>2</td>
<td>50%</td>
<td>1.8</td>
</tr>
<tr>
<td>0.75</td>
<td>300</td>
<td>4</td>
<td>0</td>
<td>0%</td>
<td>2.8</td>
</tr>
<tr>
<td>0.75</td>
<td>500</td>
<td>3</td>
<td>1</td>
<td>25%</td>
<td>3.1</td>
</tr>
</tbody>
</table>

CRRA denotes “Coefficient of Relative Risk-aversion.”

Table 4 shows the calculated coefficients of relative risk aversion required for subjects whose values are greater than the cost to vote no. Also, the number of observations who vote yes and no are reported in the third and fourth columns in Table 4. For example, the second row shows that a coefficient of relative risk aversion greater than 0.7 induces a no vote for subjects who participate in the FOPR treatment of \( p = 0.25 \) and whose difference between their values and the cost is 100JPY (1USD). This implies that voting no by three subjects in this treatment is consistent with choices made by agents whose coefficient of relative risk aversion is greater than 0.7, whereas voting yes by one subject is consistent with choices made by agents whose coefficient of relative risk aversion is less than 0.7. As an extreme example, the no vote made by the subject who participates in the treatment of \( p = 0.75 \) and whose value and cost difference is 500JPY (5USD) is consistent with choices made by agents whose coefficient of relative risk aversion is greater than 3.1. The distribution of relative risk aversion coefficients we report here is fairly consistent with the empirical parameter estimates of individual relative risk aversion (Szpiro, 1986; Carlsson et al., 2005; Chetty, 2006; Chetty and Szeidl, 2007).
The analysis in this section shows that whether the subjects whose values are greater than the cost in the FOPR design violate the IC1 or not could depend on degree of risk-aversion, magnitude of difference between their value and the cost, and the probability of the referendum being binding. Subjects who have a higher coefficient of relative risk aversion are more likely to violate the IC1. The higher probability needs the higher coefficient of relative risk aversion (more risk averse) for subjects to vote no, and subjects with a bigger difference between their values and the cost need a higher coefficient of relative risk aversion (more risk averse) to vote no. In other words, the higher probability (more consequential) and the bigger difference imply less violation of the IC1 due to risk aversion.

5. Concluding Remarks

In this paper, we examine the incentive properties of probabilistic referenda. We first extend the outcome space from two to four possible outcomes. Then, we provide a closer look at and theoretical analysis of the incentive compatibility of probabilistic referenda. Finally, we conduct an induced-value experimental test of our theoretical predictions and gain new insights that are contrary to results from previous studies. First, our results in the standard perfectly correlated induced value experiments confirm previous findings of probabilistic referenda with the exception at the probability 0.01. This suggests that some subjects reacted differently to very small probabilities and did not satisfy the first incentive compatibility (IC1). Second, our results suggest that moving away from perfectly correlated prices and quantities undermines the incentive compatibility result found in other studies. Third, our experimental results are consistent with choices made by

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17 Our observations reported in Table 4 are consistent with risk aversion. However, we should note that we could not mention that their voting decisions are made because of risk aversion.
risk-averse agents in our theoretical analysis. Our results suggest that a negative hypothetical bias possibly occurs even in consequential probabilistic referenda if there are four possible outcomes in respondents’ cognitive processes and respondents have concave utility functions, implying that dichotomous choice CV possibly underestimates true values.

We confirm that if CV referenda are consequential and there are only two outcomes, in other words payment and provision are perfectly correlated, then CV referenda would be incentive compatible. This implies that every CV survey should be designed such that it is consequential. In addition, researchers need to pay attention to the possible outcomes that respondents might have. If respondents think about more than two perfectly correlated outcomes (for example, as the result of the referendum respondents might have to pay the cost of the project but the policy makers might not conduct the project, or the reverse), then CV referenda are not incentive compatible (in terms of IC1) and this may result in biased estimates. This new perspective will be more important in applications where there exists uncertainty over the cost and/or provision, like ecosystem restoration projects and climate change policy.

We close the paper with a brief guideline for eliciting true value implicated by our results. First of all, basically all consequential binary probabilistic referenda should satisfy the second incentive compatibility (IC2), which implies that the mechanisms are truth revealing in the sense that no subject has an incentive to lie to reveal his/her preference. However from the perspective of researchers, it is ideal if probabilistic referenda satisfy IC1 because not only the referendum mechanism guarantees that the referendum result is based on truth revelations but also researchers or policy makers can estimate the distribution of true values using only information about voting decisions and the costs.
We find that the TOPR satisfy IC1, implying that the necessary information for estimating the distribution of true values is only voting decisions and the costs. On the other hand, the FOPR analyzed in this paper, in which the probability of payment is equal to the probability of provision, satisfy the IC2 but not the IC1 although for risk neutral respondents, the FOPR satisfy the IC1. Our analysis of risk aversion in section 4 shows that respondents whose values are greater than the costs could vote no probably because of risk aversion. In addition, respondents who have higher risk aversion are more likely to vote no, and this tendency could become stronger as the subjective consequentiality (i.e. the probability of the referendum being binding) and/or the difference between the value and cost decrease. These results imply that using only information about voting decisions and the costs could cause underestimation of true values if respondents are risk averse as previous empirical findings support that most people are risk averse over modest stakes (Szpiro, 1986; Carlsson et al., 2005; Chetty and Szeidl, 2007). In this case, to estimate the distribution of true values, we would need additional information about the measure of respondent’s individual risk attitude and the subjective probability of the referendum being binding.

Though we treat only the case that the probability of payment (payment uncertainty) is equal to the probability of provision (provision uncertainty) in this paper, if these probabilities are different from each other then we would need to measure these probabilities in addition to subject’s risk attitude. Mitani and Flores (2010) deal with the case in which the probability of payment is not equal to the probability of provision, using a threshold provision mechanism. Their experimental analysis suggests that the relative importance between payment and provision uncertainty has a huge impact on the payment decision. Off diagonal FOPR in which the probability of payment is not always equal to the probability of provision will be explored in future research.
In short, satisfying the IC1 allows us to ignore the influence of the probabilistic nature and the subject’s risk attitude toward the uncertainty. This implies that it is important for probabilistic referenda mechanisms to satisfy the IC1 from the perspective of eliciting individual preferences for public projects. This finding would be important for survey design in practice as well as theoretical aspect of CV referenda.

References


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