PHY 889: TOPOLOGICAL CONDENSED MATTER PHYSICS
Winter 2021

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Lectures: Thursdays, 10 a.m. - noon; Fridays: 2 p.m. - 4 p.m., via Microsoft Teams

I. INTRODUCTION

The development of quantum physics in the beginning of the 20th century revolutionized people's understanding of materials. Based on the laws of quantum physics, solids were classified into insulators, semiconductors, metals, and superconductors. For decades, it was believed that all insulators were similar to one another when it came to their inability to conduct electricity, and that all superconductors were also similar to one another when it came to their extreme ability to carry electrical current.

Such belief was first shaken in 1980, and then shattered in 2005, when it became apparent that the behavior of electrons in solids can also be classified using a branch of mathematics known as topology. In particular, the electronic energy bands and wave functions are characterized by integers known as topological invariants. These invariants manifest themselves physically by virtue of peculiar electronic states localized at sample boundaries.

For example, according to the topological classification of solids, some insulators (dubbed "topological" insulators) conduct electricity on their surfaces whereas others (dubbed "non-topological" or "ordinary" insulators) do not. The metal at the surface of a topological insulator is quite special: electrons behave as relativistic Dirac fermions, their conduction of electricity is remarkably robust, and their magnetic properties peculiar.

Aside from being a remarkable feat of fundamental science (awarded with the 2016 Nobel prize), the advent of topological materials harbors a promise for practical applications. For example, the discovery of Majorana zero modes in topological superconducting devices is a stepping stone towards fault-tolerant qubits and low-decoherence quantum computers. In addition, the surface states of topological insulators may lead to low-dissipation spintronics devices.

II. COURSE OUTLINE

This graduate level course will present an introduction to topological insulators, semimetals and superconductors at a level that is accessible both for MSc and PhD students. The prerequisites for this course are a basic knowledge of Solid State Physics (familiarity with Bloch's theorem, Brillouin zones, energy bands, basic semiconductor physics) and Quantum Mechanics (familiarity with the concept of eigenstates, eigenvalues, Hamiltonians). At the end of the course you will have hopefully gained a solid understanding of the physics that underlies these interesting materials and you will be prepared to read the current literature and conduct research in the field.

The topics covered will be the following:

- **1. - Elements of topological band theory:**

  1.1) Berry phase, Berry curvature, Chern number.

- **2. - Topological band theory in one dimension**:

  2.1) Berry phase theory of the electrical polarization.

  2.2) Thouless charge pump and the Chern number.

  2.3) Su-Schrieffer-Heeger model. Dirac fermions and the Jackiw-Rebbi zero modes.

  2.4) Kitaev model for 1D topological superconductors. Majorana zero modes.

- **3. - Topological band theory in two dimensions**:

  3.1) (Integer) quantum Hall effect.

  3.2) Link between the Chern number and the quantized Hall conductivity.

  3.3) Link between the Chern number and the gapless chiral edge states.

  3.4) Link between the Thouless charge pump and the Laughlin argument.

3.5) Graphene.

3.6) Haldane/Chern *vs.* Semenoff insulator in graphene.

3.7) The Kane-Mele model for graphene and the quantum spin Hall effect.

3.8) $\mathbb{Z}_2$ topological invariant.

3.9) Quantum spin Hall effect in HgTe/CdTe quantum wells.

3.10) Helical and chiral topological superconductors. Majorana edge states.

- **4.- Topological band theory in three-dimensions**:

(4.1) Semiclassical equations of motion for electrons in crystals.

(4.2) Three dimensional quantum Hall effect.

(4.3) $\mathbb{Z}_2$ invariants of 3D insulators with time-reversal symmetry.

(4.4) Surface states of 3D topological insulators.

(4.5) Model Hamiltonian of the simplest 3D topological insulator: Bi_2Se_3.

(4.6) Axion electrodynamics and its applications.

(4.7) Topological crystalline insulators.

(4.8) Topological superconductivity in three dimensions.

(4.9) Topological semimetals: Weyl fermions and beyond.

- **5.- Possible extra topics (time-permitting; student suggestions welcome)**

(5.1) Higher order topological phases.

(5.2) Topological phases in metamaterials (photonics, acoustics, mechanics, microwaves).

(5.3) Topological phases in the presence of strong interactions.

III. TEXTBOOK AND GENERAL REFERENCES

Since the subject of topological condensed matter physics is rather new, there are not many textbooks available. Here are three that I will use in this course:

- *Topological Insulators and Superconductors*, B.A. Bernevig and T.L. Hughes (Princeton University Press, 2013).
- *Topological Insulators: Dirac Equations in Condensed Matters*, S.-Q. Shen (Springer, 2013).
- *Berry phases in electronic structure theory*, D. Vanderbilt (Cambridge University Press, 2018).

A copy of Bernevig's book is available in the library. Also, as a UdeS student you can download Shen's book and Vanderbilt's book for free (search for them in the online library catalogue).

IV. ASSIGNMENTS AND GRADING

Several problem sets will be assigned during the course (for a total of 50% of the final grade). Working out the assignments will be key to better absorb and understand the content of the course. There will be no exams; instead, you will give a 20-minute presentation at the end of the session on a selected topic of your interest (40% of the final grade).

I would like this course to be a good learning experience for everyone (including myself!). To that end, I invite an open and participative atmosphere, where everyone is welcome to ask questions and make comments. I also welcome questions and discussions off class. 10% percent of the final grade will be alloted to participation.